Activity 32 Combining transformations

Aim: Explore and describe examples of linear transformations

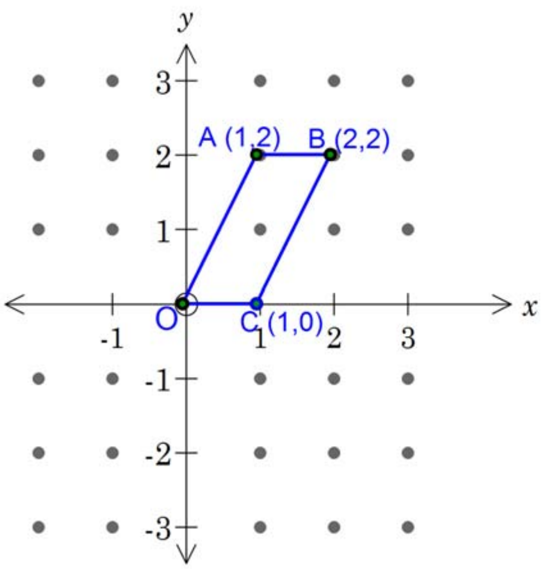
The linear transformations T_1 to T_6 are defined as

$$\mathbf{T}_1=\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix},\ \mathbf{T}_2=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix},\ \mathbf{T}_3=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix},\ \mathbf{T}_4=\begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix},\ \mathbf{T}_5=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\ \mathbf{T}_6=\begin{bmatrix}0.6 & 0.8\\ -0.8 & 0.6\end{bmatrix}$$

1. Write a 2×4 matrix **P** to represent the vertices of parallelogram OABC drawn in Q2 below.

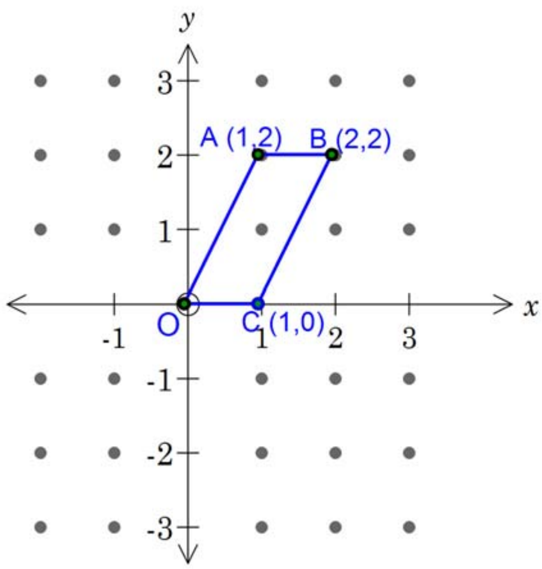
2. For each transformation matrix $\mathbf{T}_1$ to $\mathbf{T}_6$:
   - Calculate the determinant.
   - Calculate $\mathbf{T}_i\mathbf{P}$.
   - Sketch the image of the parallelogram.
   - Describe the transformation.

a) Det($\mathbf{T}_1$) : $\mathbf{T}_1\mathbf{P}$:



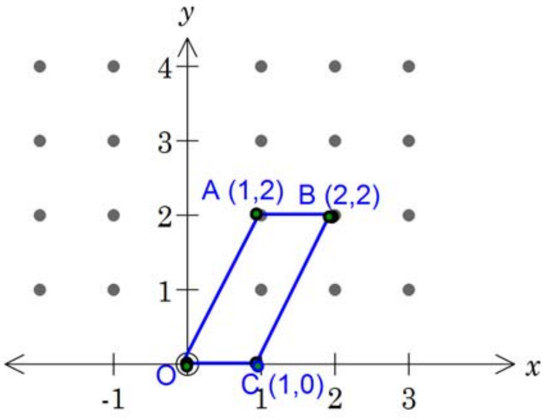
Description:

b) Det($\mathbf{T}_2$) : $\mathbf{T}_2\mathbf{P}$:



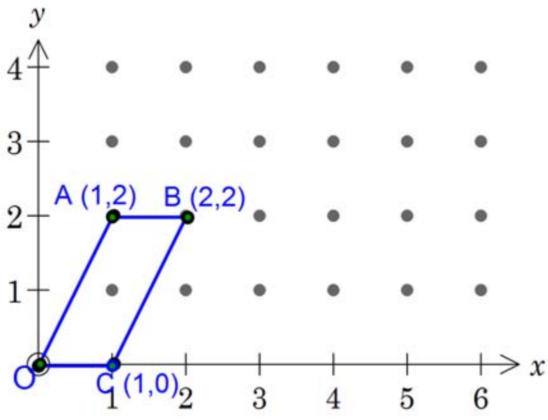
Description:

c) Det($\mathbf{T}_3$) : $\mathbf{T}_3\mathbf{P}$:



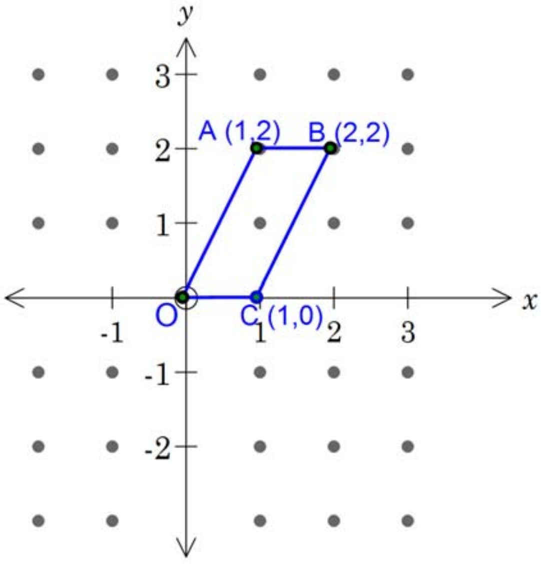
Description:

d) Det($\mathbf{T}_4$) : $\mathbf{T}_4\mathbf{P}$:



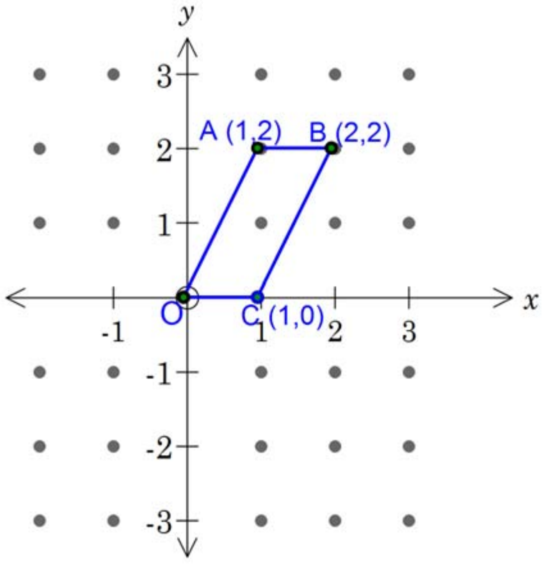
Description:

e) Det($\mathbf{T}_5$) : $\mathbf{T}_5\mathbf{P}$:



Description:

f) Det($\mathbf{T}_6$) : $\mathbf{T}_6\mathbf{P}$:



Description:

3.

a) Given $\mathbf{T}_7 = \mathbf{T}_1\mathbf{T}_5$, calculate $\mathbf{T}_7$ and describe the transformation.

b) Given $\mathbf{T}_8 = \mathbf{T}_5\mathbf{T}_1$, calculate $\mathbf{T}_8$ and describe the transformation.

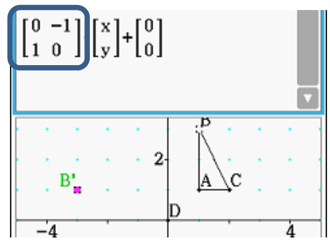
c) Are $\mathbf{T}_7$ and $\mathbf{T}_8$ equivalent? Explain your answer.

d) Given $\mathbf{T}_9 = (\mathbf{T}_2)^4$, calculate $\mathbf{T}_9$ and describe the transformation.

4. Determine the transformation matrices for each transformation.

In the geometry window

- Select a point
- Select [Draw | Construct | and the appropriate transformation]
- Select [View | Zoom to Fit] if required
- Select the point and image point
- Drag into the Main window
- Read the 2 x 2 transformation matrix



a) A dilation, scale factor 5.

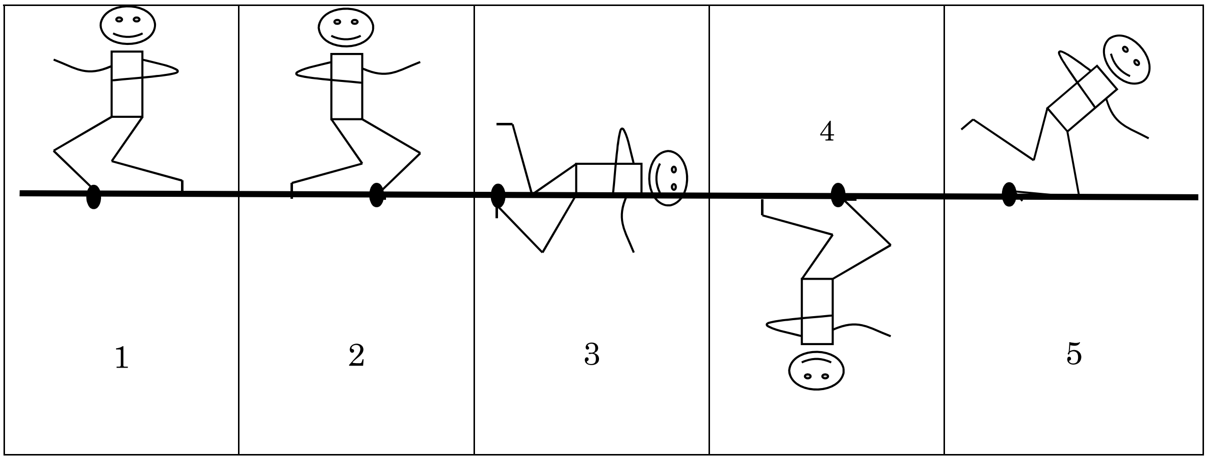
b) A rotation of $120°$ clockwise about the origin.

c) A rotation of $180°$ anticlockwise about the origin.

d) A reflection in the line $y = -x$.

e) A reflection in the line $y = \sqrt{3}x$.

5.



This acrobat is on the high wire. Use the dot (foot position) as the position that remains in the same place (origin) in each frame.

a) Find the transformation matrix which takes the figure to the next frame.
(From Frame 4 to 5 requires more than one simple transformations.)

1→2	2→3	3→4	4→5

b) Check your answers by applying the transformations to a simpler figure using your ClassPad.

Learning notes

Q1 Each point is represented as a column.

Q2 You can store the matrices as T1, T2 etc. They will then be easy to recall for later calculations.

Q5 Once you have identified the transformation as, for example, a rotation of ... you can then apply that transformation in the Geometry application. Dragging a point and its image into Main will display the transformation matrix.